STAT 2290 Homework 2 Solutions

Problem 1. (Review)

Scores on a particular test follow a normal distribution with a mean of 75.6 and a standard deviation of 7.8.

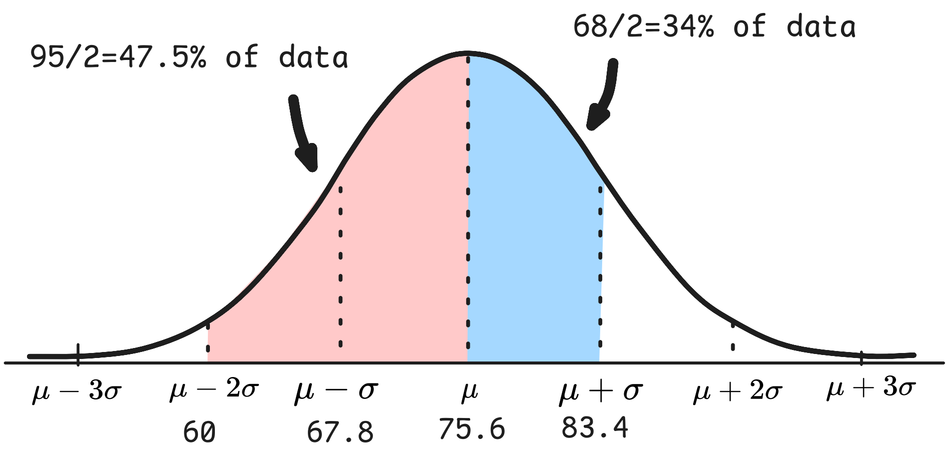
(a) What percent of students score between 60 and 83.4?

(b) Find the score separating the top 16% of the scores from the bottom 84% of scores.

(c) What percent of students score below 60?

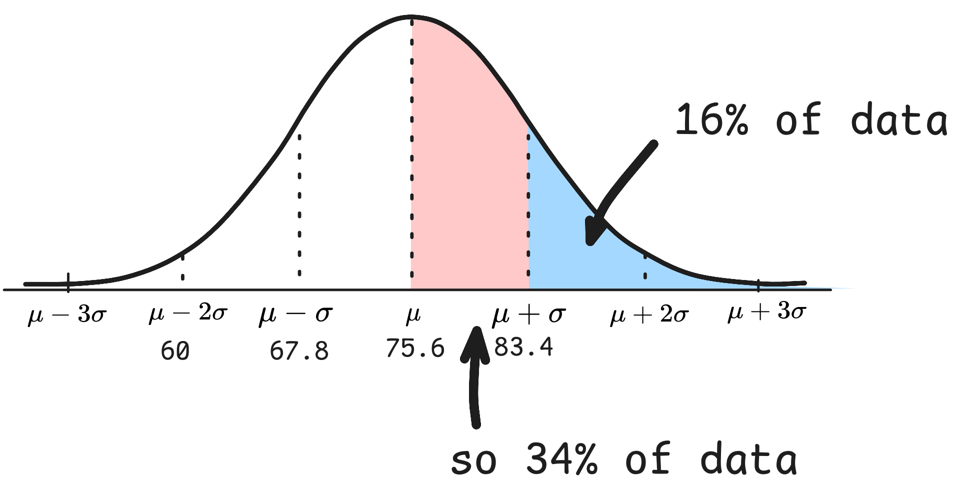
Answer to (a):

- $60 = 75.6 - 7.8 \times 2$ is two standard deviations below the mean, so by the 68-95-99.7 Rule, there is $95/2 = 47.5\%$ of data between 60 and 75.6.
- $83.4 = 75.6 + 7.8 \times 1$ is one standard deviation above the mean, so by the 68-95-99.7 Rule, there is $68/2 = 34\%$ of data between 75.6 and 83.4.
- Together, there are $47.5 + 34 = 81.5\%$ of data between 60 and 83.4.



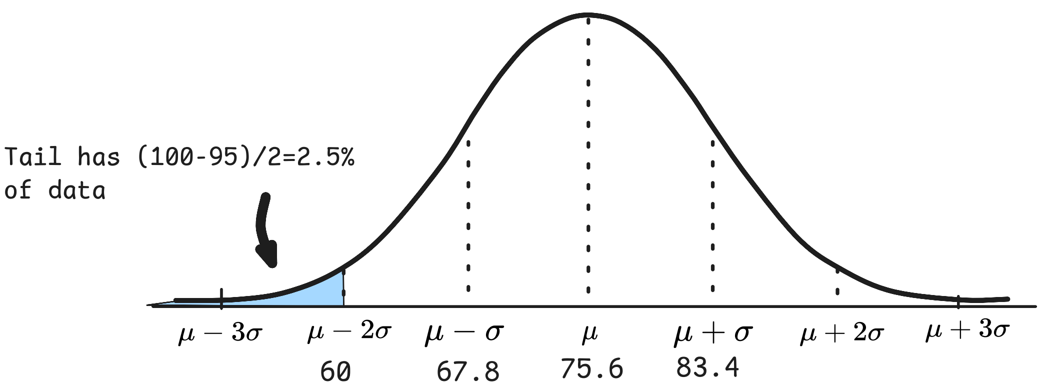
Answer to (b):

- 50% of the scores are above the mean, so $50 - 16 = 34\%$ of the score are above the mean but below the top 16%. Since $68 = 34 \times 2$, the top 16% mark sits at one standard deviation above the mean. The score separating the top 16% of the scores from the bottom 84% of scores is $75.6 + 7.8 = 83.4$.



Answer to (c):

- $60 = 75.6 - 7.8 \times 2$ is two standard deviations below the mean, so by the 68-95-99.7, $100 - 95 = 5\%$ of data are in the two tails beyond two standard deviations out, and so $5/2 = 2.5\%$ of data are in the lower tail two standard deviations out.



Problem 2. (Basic probability)
A coin is flipped three times. We wish to find the probability of getting at least two heads.
(a) Identify the sample space S of interest such that each outcome occurs with the same probability. List out all elements of S.
(b) Identify the event E of interest. List out all elements of E.
(c) Find the probability $P(E) = |E|/|S|$.

Answer to (a):

- $\{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

Answer to (b):

- $\{HHH, HHT, HTH, THH\}$

Answer to (c):

- $4/8$

Problem 3. (Basic probability)
In a poker hand consisting of 6 cards, we wish to find the probability of holding exactly 3 aces.
(a) Identify the sample space S of interest such that each outcome occurs with the same probability. Do not need to list out all elements of S.
(b) Identify the event E of interest. Do not need to list out all elements of E.
(c) Find the probability $P(E) = |E|/|S|$.

Answer to (a):

- Set of 6 cards chosen without replacement from a standard deck of cards.

Answer to (b):

- Set of 6 cards chosen without replacement from a standard deck of cards, that have exactly 3 aces and 3 non-aces.

Answer to (c): We must draw 3 aces from the 4 aces, and fill the remaining 3 cards in our hand from the 48 non-aces, so the probability is:

$$\frac{\binom{4}{3}\binom{48}{3}}{\binom{52}{6}}$$

Problem 4. (Basic probability)
In a poker hand consisting of 6 cards, find the probability of holding 4 hearts and 2 clubs.

Answer: We must choose 4 hearts from the 13 hearts and 2 clubs from the 13 clubs in the standard deck of cards, so the probability is:

$$\frac{\binom{13}{4}\binom{13}{2}}{\binom{52}{6}}$$

Problem 5. (Conditional probability)
I draw 4 cards without replacement from a standard deck of 52 playing cards. I tell you that all my cards are either 10's, J's, Q's, K's, or A's (or a mix of these cards). Given this information, what is the probability that I do not have any 10's or A's in my hand of 4 cards?

We view this conditional probability as reducing our sample space down from drawing out of the 52 cards, to drawing out of 10's, J's, Q's, K's, or A's ($5 \times 4 = 20$ cards overall). So our question is now: suppose we draw from these 20 cards. What is the probability that we draw only J's, Q's, K's to get a 4-card hand?

The overall ways to draw any 4 of the 20 cards is

$$\binom{20}{4} = \frac{20 \times 19 \times 18 \times 17}{4 \times 3 \times 2} = 5 \times 19 \times 3 \times 17$$

But if we want to draw from J's, Q's, K's (12 cards), then there are

$$\binom{12}{4} = \frac{12 \times 11 \times 10 \times 9}{4 \times 3 \times 2} = 11 \times 5 \times 9$$

ways.

The desired probability is therefore:

$$\frac{\binom{12}{4}}{\binom{20}{4}} = \frac{33}{19 \times 17}$$

You don't need to simplify: just leave your answer as $\frac{\binom{12}{4}}{\binom{20}{4}}$.

Answers:
P1(a): 81.5%
P1(b): 83.4
P1(c): 2.5%
P2(c): 1/2
P3(c): ≈ 0.0034
P4: ≈ 0.0027
P5: $33/323 \approx 0.102$